

# Transition Distribution Amplitudes for $\gamma^*\gamma$ collisions

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## Abstract

We study the exclusive production of  $\pi\pi$  and  $\rho\pi$  in hard  $\gamma^*\gamma$  scattering in the forward kinematical region where the virtuality of one photon provides us with a hard scale in the process. The newly introduced concept of Transition Distribution Amplitudes (TDA) is used to perform a QCD calculation of these reactions thanks to two simple models for TDAs. The sizable cross sections for  $\rho\pi$  and  $\pi\pi$  production may be tested at intense electron-positron colliders such as CLEO and B factories (Belle and BABAR).

## 1 Introduction

In a series of recent papers [1], we have advocated that factorisation theorems [2] for exclusive processes may be extended to the case of other reactions such as  $\pi^-\pi^+ \rightarrow \gamma^*\gamma$  and  $\gamma_L^*\gamma \rightarrow AB$  in the kinematical regime where the virtual photon is highly virtual (of the order of the energy squared of the reaction) but the momentum transfer  $t$  is small. This enlarges the successful description of deep-exclusive  $\gamma\gamma$  reactions in terms of distribution amplitudes [3] and/or generalised distribution amplitudes [4] on the one side and perturbatively calculable coefficient functions describing hard scattering at the partonic level on the other side. The reactions

$$\gamma_L^*\gamma \rightarrow \rho^\pm\pi^\mp, \quad \gamma_L^*\gamma \rightarrow \pi^\pm\pi^\mp, \quad \gamma_L^*\gamma \rightarrow \pi^0\pi^0,$$

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in the near forward region and for large virtual photon invariant mass  $Q$ , may be studied in detail at intense electron colliders such as those which are mostly used as B factories.

With the kinematics described in Fig. 1, we define the  $\gamma \rightarrow \pi$  transition distribution amplitudes (TDAs)  $T(x, \xi, t)$  as the Fourier transform of matrix elements  $\langle \pi(p_\pi) | \mathcal{O} | \gamma(p_\gamma) \rangle$  where  $\mathcal{O} = \bar{\psi}(\frac{-z}{2})[\frac{-z}{2}, \frac{z}{2}] \Gamma \psi(\frac{z}{2})$  with  $\Gamma = \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}$ . The Wilson line  $[\frac{-z}{2}, \frac{z}{2}]$  provides the QCD-gauge invariance for non-local operators and equals unity in a light-like (axial) gauge. We do not write the electromagnetic Wilson line caused by the presence of the photon, since we choose an electromagnetic axial gauge for the photon. We then factorise the amplitude of the process  $\gamma_L^*\gamma \rightarrow A\pi$  as

$$\int dx dz \Phi_A(z) M_h(z, x, \xi) T(x, \xi, t), \quad (1)$$

with a hard amplitude  $M_h(z, x, \xi)$  and  $\Phi_A(z)$  is the hadron A distribution amplitude (DA).

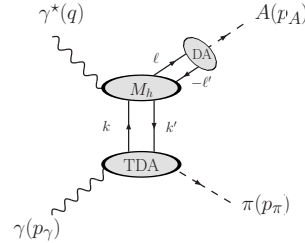


Figure 1: The factorised amplitude for  $\gamma^*\gamma \rightarrow A\pi$  at small transfer momentum.

The variable  $z$  is as usual the light-cone

momentum fraction carried by the quark entering the meson  $A$ ,  $x + \xi$  (resp.  $x - \xi$ ) is the corresponding one for the quark leaving (resp. entering) the TDA. The skewness variable  $\xi$  describes the loss of light-cone momentum of the incident photon and is connected to the Bjorken variable  $x_B$ .

Contrarily to the case of generalised parton distributions (GPD) where the forward limit is related to the conventional parton distributions measured in the deep inelastic scattering (DIS), there is no such interesting constraints for the new TDAs. The only constraints are sum rules obtained by taking the local limit of the corresponding operators and possibly soft limits when the momentum of the meson in the TDA vanishes. Lacking any non-perturbative calculations of matrix element defining TDAs, we are forced to build toy models to get estimates for the cross sections, to be compared with future experimental data.

For definiteness, let us consider the  $\gamma \rightarrow \pi^-$  vector TDAs which is given by ( $P = \frac{p_{\pi^-} + p_\gamma}{2}$ ,  $\Delta = p_{\pi^-} - p_\gamma$ ):

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^- | \bar{d}(\frac{-z}{2}) \left[ \frac{-z}{2}; \frac{z}{2} \right] \gamma^\mu u(\frac{z}{2}) | \gamma \rangle = \frac{1}{P^+} \frac{i e}{f_\pi} e^{\mu \varepsilon P \Delta_\perp} V^{\pi^-}(x, \xi, t).$$

A sum rule may be derived for this photon to meson TDA. Since the local matrix element appears in radiative weak decays, we can relate it to the form factor  $F_V$ :

$$\int_0^1 dx V^{\pi^\pm}(x, \xi, t) = \frac{f_\pi}{m_\pi} F_V^{\pi^\pm}(t), \quad (2)$$

with  $F_V^{\pi^\pm} = 0.017 \pm 0.008$ .

Let us consider the  $\rho_L^\pm \pi^\mp$  production case when the  $\rho$  flies in the direction of the virtual photon and the  $\pi$  enters the TDA. Note that in the neutral  $\rho^0 \pi^0$  case, the TDA process is forbidden by  $C$ -conjugation.

For definiteness, we choose, in the CMS of the meson pair,  $p = \frac{Q^2 + W^2}{2(1+\xi)W}(1, 0, 0, -1)$  and

$n = \frac{(1+\xi)W}{2(Q^2+W^2)}(1, 0, 0, 1)$  and we express the momenta through a Sudakov decomposition:

$$\begin{aligned} p_\gamma &= (1 + \xi)p, \\ q &= \frac{Q^2 + W^2}{1 + \xi}n - \frac{Q^2}{Q^2 + W^2}(1 + \xi)p, \\ p_{\pi^-} &= (1 - \xi)p - \frac{\Delta_T^2}{1 - \xi}n + \Delta_T, \end{aligned}$$

with  $\Delta_T^2 = \frac{1-\xi}{1+\xi}t$ .

We can see that  $\xi$  is determined by the external kinematics through  $\xi \simeq \frac{Q^2}{Q^2 + 2W^2} -$  similarly to  $x_B = \frac{Q^2}{Q^2 + W^2}$  to which it is linked via the simple relation  $\xi \simeq \frac{x_B}{2 - x_B}$ .

The amplitude  $\mathcal{M}_{\gamma^* \gamma}^{TDA}(Q^2, \xi, t)$  for the reaction  $\gamma_L^* \gamma \rightarrow \rho_L^+ \pi^-$  is proportional to  $V(x, \xi, t)$ ; it reads

$$- \int_{-1}^1 dx \int_0^1 dz \frac{f_\rho}{f_\pi} \phi_\rho(z) M_h(z, x, \xi) V(x, \xi, t),$$

where the hard amplitude  $M_h(z, x, \xi)$  is ( $\bar{z} = 1 - z$ ):

$$\frac{8\pi^2 \alpha \alpha_s C_F}{N_C Q z \bar{z}} \left( \frac{Q_u}{x - \xi + i\epsilon} + \frac{Q_d}{x + \xi - i\epsilon} \right) \epsilon^{n \varepsilon p \Delta}.$$

We choose  $\phi_\rho(z) = 6z\bar{z}$  as the asymptotic normalised meson distribution amplitude and  $f_\rho = 0.216$  GeV. After separating the real and imaginary parts of the amplitude, the  $x$ -integration gives:

$$\begin{aligned} & \int_{-1}^1 dx \left( \frac{Q_u}{x - \xi + i\epsilon} + \frac{Q_d}{x + \xi - i\epsilon} \right) V(x, \xi, t) \\ &= Q_u \int_{-1}^1 dx \frac{V(x, \xi, t) - V(\xi, \xi, t)}{x - \xi} \\ &+ Q_d \int_{-1}^1 dx \frac{V(x, \xi, t) - V(-\xi, \xi, t)}{x + \xi} \\ &+ Q_u V(\xi, \xi, t) \left( \log \left( \frac{1 - \xi}{1 + \xi} \right) - i\pi \right) \\ &+ Q_d V(-\xi, \xi, t) \left( \log \left( \frac{1 + \xi}{1 - \xi} \right) + i\pi \right). \end{aligned}$$

The scaling law for the amplitude is

$$\mathcal{M}_{\gamma^* \gamma}^{TDA}(Q^2, \xi, t) \sim \frac{\alpha_s \sqrt{-t}}{Q}, \quad (3)$$

up to logarithmic corrections due to the anomalous dimension of the TDA.

## 2 Models for TDAs

To quantify the magnitudes of the cross section, we need to adopt a specific model for the non perturbative TDA. As a first choice, we start from a double distribution for  $t = 0$  [5] which leads to a  $x$ - and  $\xi$ -dependence of the TDA in the form  $V^{(0)}(x, \xi)$ :

$$\int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi\alpha) f(\beta, \alpha),$$

with  $f(\beta, \alpha) = q(\beta)h(\beta, \alpha)$ , where  $q(\beta)$  is analogous to the forward quark distribution in the GPD case and  $h(\beta, \alpha)$  is a profile function parametrised as

$$\frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|\beta|)^2 - \alpha^2]^b}{(1-|\beta|)^{2b+1}},$$

where the parameter  $b$  characterises the strength of the  $\xi$ -dependence. As a first guess, we assume that the  $\beta$ -dependence of  $q$  is given by a simple linear law  $q(\beta) = 2(1-\beta)\theta(\beta)$  and we assume a mild  $\xi$  dependence as given by  $b = 1$ . Moreover we implement the normalisation (with  $\int dx G^{(0)}(x, \xi) = 1$ ) and the  $t$ -dependence of the TDA through the vector form factor:

$$V(x, \xi, t) = V^{(0)}(x, \xi) \cdot \frac{f_\pi}{m_\pi} F_V(t).$$

The  $t$ -dependence of this form factor has been studied in chiral perturbation theory and turned out to be weak, so we shall neglect it in this model and we shall use the measured values at  $t=0$ .

As a second model, we use the initial  $t$ -dependent double distribution of Tiburzi [6]. Explicitly,  $V(x, \xi, t)$  is written as

$$\frac{f_\pi}{m_\pi} \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi\alpha) \times \frac{m^2}{4\pi^2(m^2 + \frac{m_\pi^2\beta(\alpha+\beta-1)}{2} - \frac{t(1-\alpha^2-\beta(2-\beta))}{2})}$$

where  $m$  is set to 0.18 GeV [6]. The sum rule is satisfied for  $t = -0.5 \text{ GeV}^2$ . We shall keep

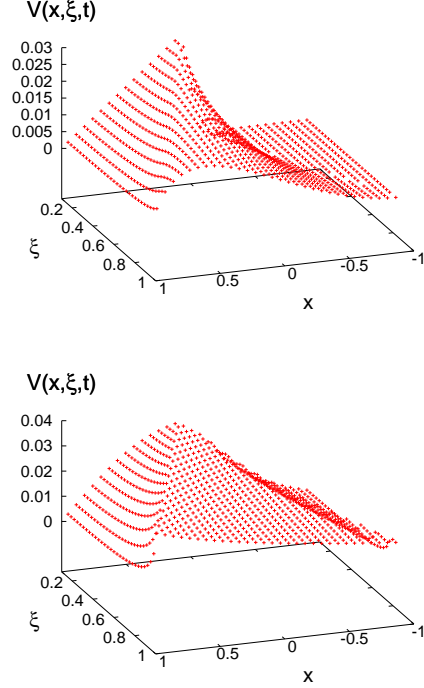


Figure 2: The  $\gamma \rightarrow \pi^-$  vector transition distribution amplitude  $V(x, \xi, t)$  in Model 1 and in Model 2 (for  $t = -0.5 \text{ GeV}^2$ ).

$t$  fixed in the following to enable comparison with Model 1. In the following, we shall refer to this approach as Model 2. We see that these two models give quite different results and we shall use both in section 3 to estimate the sensitivity of the cross sections to TDA models. Other models for TDAs have recently been worked out [7] and models for pion GPDs [8] could also be applied to the TDA case.

## 3 Results and conclusions

Since we want to focus on the study of the TDA behaviour, we decide to choose  $Q^2$ ,  $t$ ,  $\xi$  and  $\varphi$  as our kinematical variables. The

differential cross section reads

$$\frac{d\sigma^{e\gamma \rightarrow e\rho\pi}}{dQ^2 dt d\xi d\varphi} = \frac{|\mathcal{M}^{e\gamma \rightarrow e\rho\pi}|^2}{32(2\pi)^4 s_{e\gamma}^2 \xi(\xi+1)}.$$

In Fig.3 (a), we plot the cross section (after a trivial  $\varphi$  integration) vs  $\xi$  and in Fig.3 (b) vs  $Q^2$  for both models of the vector TDA. The behaviour for intermediate values of  $\xi$  is sensitive to specific models for the TDA. As shown in 3 (b), the  $Q^2$ -behaviour is model independent and thus constitutes a crucial test of the validity of our approach.

Both real and imaginary part of the amplitude contribute significantly to the cross section, which is reasonable at these moderate energies. Since the phenomenological analysis of the pion form factor indicates that a rather large value of  $\alpha_s$  should be used together with the asymptotic DA, we use  $\alpha_s = 1$  in our numerical study. Our conclusions would not be strongly affected by a different choice.

It is now time to test experimentally the new factorised QCD approach to forward hard exclusive scattering in  $\gamma^*\gamma$  exclusive reactions. We believe that our models for the photon to meson transition distribution amplitudes are sufficiently constrained to give reasonable orders of magnitude for the estimated cross sections. Cross sections are large enough for quantitative studies to be performed. After verifying the scaling of the cross sections, one should be able to measure these new hadronic matrix elements, and thus open a new gate to the understanding of the hadronic structure.

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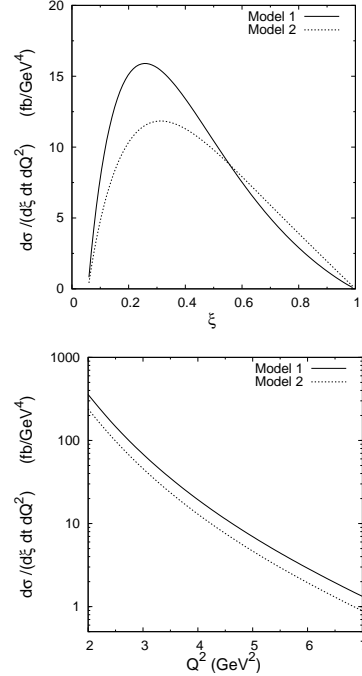


Figure 3:  $e\gamma \rightarrow e'\rho_L^+\pi^-$  differential cross section plotted as a function of  $\xi$  (a) and  $Q^2$  (b) for  $s_{e\gamma} = 40 \text{ GeV}^2$ ,  $t = -0.5 \text{ GeV}^2$  and respectively  $Q^2 = 4 \text{ GeV}^2$  for (a) and  $\xi = 0.2$  for (b).

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